JEE-Main-30-01-2024 (Memory Based) [EVENING SHIFT]

Mathematics

Question: Bag A 3 write, 7 Red Ball Bag B → 3 white, 2 red if white ball is found find P of its being from Bag A.

Answer: $\frac{1}{3}$

Solution:

$A \to 3W, TR$

$B \to 3W, 2R$

$P(W) = \frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{3}{5}$

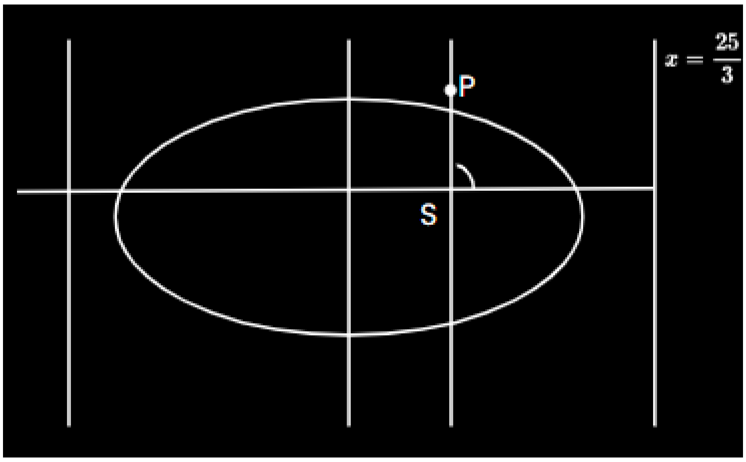
$P\left(\frac{A}{W}\right) = \frac{\frac{3}{20}}{\frac{3}{20} + \frac{6}{20}} = \frac{1}{3}$

Question: 5x + 7y = 50

A(A, B), B(O, B) lie in this P divides AB in ratio 7 : 3. $3x - 25 = 0$ is directrix of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with S as its focus on side of directrix if P lies on perpendicular form S to X axis, then find LR of ellipse

Answer: 6.4

Solution:



$5x + 7y = 50$

$A(10,0), B\left(0, \frac{50}{7}\right)$

$P:(3,5)$

$x=\frac{25}{3}=\frac{a}{e}$

$ae=3;\ a^2=245$

$e=\frac{3}{5}$

LR $=\frac{2b^2}{a}=2a\left(1-e^2\right)$

$=10\left(\frac{16}{25}\right)=\frac{32}{5}=6.4$

Question: $A:\frac{x^2}{4}-\frac{x^2}{9}=1$. If P is point on and area of $\Delta PSS'$; is $2\sqrt{13}$ (S,S' is focus) find square of distance of P from origin.

Answer: $\frac{52}{9}$

Solution:

$a=2, b=3$

$e=\sqrt{1+\frac{9}{4}}=\frac{\sqrt{13}}{2}$

$SS'=2ae$

$=2\sqrt{13}$

Area $=\frac{1}{2}\times 2\sqrt{13}\times|\beta|=2\sqrt{3}$

$\Rightarrow |\beta|=2$

$\frac{\alpha^2}{4}=1+\frac{4}{9}=\frac{13}{9}$

$\alpha^2+\beta^2=\frac{52}{9}+4=\frac{88}{9}$

$\alpha^2=\frac{52}{9}$

Question: $f(x)=\frac{x}{\left(1+x^4\right)^{\frac{1}{4}}}; g(x)=f\left(f\left(f\left(f(x)\right)\right)\right)$. Find $\int_0^{\sqrt{2\sqrt{5}}} x^2 g(x)\,dx$

Answer: $\frac{13}{6}$

Solution:

$$f\left(f(x)\right)=\frac{\frac{x}{\left(1+x^4\right)^{\frac{1}{4}}}}{\left(1+\frac{x^4}{1+x^4}\right)^{\frac{1}{4}}}$$

$$=\frac{x}{\left(1+2x^4\right)^{\frac{1}{4}}}$$

$$f\left(f\left(f\left(f\left(x\right)\right)\right)\right)=\frac{x}{\left(1+4x^4\right)^{\frac{1}{4}}}$$

$$\int_0^{\sqrt{2\sqrt{5}}}\frac{x^3}{\left(1+4x^4\right)^{\frac{1}{4}}}dx=\int_1^3\frac{\frac{t^3}{4}dt}{t}\quad\begin{bmatrix}1+4x^4=t^4\\4x^3dx=t^3dt\end{bmatrix}$$

$$=\left.\frac{t^3}{2}\right|_1^3=\frac{13}{6}$$

Question: $f\left(x\right)=ae^{2x}+be^{x}+cx$

$f\left(0\right)=-1$

$f'\left(\log 2\right)=21$

$\int_0^{\log_e 4} f\left(x\right)-cx\,dx<\frac{39}{2}$ find $\left|a+b+c\right|$

Answer: 8.00

Solution:

$$f\left(0\right)=a+b=-1\quad ....(1)$$

$$f'\left(\ln 2\right)=2a\times 4+b\left(2\right)+c=21\quad ...(2)$$

$$\int_0^{\ln 4}\left(f\left(x\right)-cx\right)dx=\left[\frac{ae^{2x}}{2}+be\right]_0^{\ln 4}=8a+4b-\frac{a}{2}-b$$

$$=\frac{15a}{2}+3b=\frac{39}{2}\quad ...(3)$$

$$\frac{9a}{2}=\frac{45}{2}\Rightarrow a=5,b=-6,c=-7$$

$$\left|a+b+c\right|=\left|-8\right|=8$$

Question: Two GP, series (1), $a_1 = a$, $a_3 = b$, series (2) $b_1 = a$, $b_5 = b$. 11th from of series (1) will be which term of series (2)

Answer: 21.00

Solution:

$a_{11} = a_1 r^{10} = a\left(\frac{b}{a}\right)^5 = \frac{b^5}{a^4}$

$b_{n+1} = a \cdot \left(\frac{b}{a}\right)^{\frac{n}{4}} = \frac{b^5}{a^4}$

$\Rightarrow n = 20$

So 21th term.

Question: Given $|\vec{b}| = 2, |\vec{b} \times \vec{a}| = 2$. Then $|\vec{b} \times \vec{a} - \vec{b}|^2$ is

Options:
(a) 0
(b) 8
(c) 1
(d) 10
Answer: (a)
Solution:
$|\vec{b}| = 2,\ |\vec{b} \times \vec{a}| = 2$

$|\vec{b} \times \vec{a} - \vec{b}|^2 = |\vec{b} \times \vec{a}||\vec{b}|^2$

$4 + 4 = 8$

Question: If $f(x) = \ln\left(\frac{2x}{4x^2 - x - 3}\right) + \cos^{-1}\left(\frac{2x+1}{x+2}\right)$ if domain of $f(x)$ is $[\alpha, \beta)$ then $5\alpha - 4\beta$ is:

Options:
(a) -2
(b) 3
(c) -4
(d) 1
Answer: ()
Solution:
$\frac{2x}{2x^2 - x - 3} > 0 \Rightarrow \frac{2x}{(2x-3)(x+1)} > 0$ and $-1 \le \frac{2x+1}{x+2} \le 1$

$\Rightarrow \frac{2x+1}{x+2} - 1 \le 0$ and $\frac{2x+1}{x+2} + 1 \ge 0$

$\frac{x-1}{x+2} \le 0$ and $\frac{3x+3}{x+2} \ge 0$

$x \in (-2, 1]$ and $x \in (-\infty, -2) \cup [-1, \infty)$

$\therefore$ Domain $= (-1, 0)$

$5\alpha - 4\beta = -5$

Question: If $f(x)=(x-2)^2(x-3)^3$ and $x\in[1,4]$ of M and m denotes maximum and minimum values respectively, then $M-m$ is

Answer: 12.00

Solution:

$$f'(x)=(x-2)(x-3)^2\left[2(x-3)+3(x-2)\right]$$

$$=(x-2)(x-3)^2[5x-12]$$

$$M\Rightarrow f(2)=0 \text{ or } f(4)=2^2 1=4$$

$$\Rightarrow M=4$$

$$m\Rightarrow f\left(\frac{12}{5}\right)=0 \text{ or } f(1)=-8$$

$$-\frac{108}{325};\ m=-8$$

$$M-m=12$$

Question: Find $\vec{a}=\hat{i}+\alpha\hat{j}+\beta\hat{k},\ |\vec{b}|^2=6$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$. $\vec{a}\cdot\vec{b}=3$ then $(\alpha^2+\beta^2)|\vec{a}\times\vec{b}|^2$ is ____.

Answer: 18.00

Solution:

$$\vec{a}\cdot\vec{b}=\left|1+\alpha^2+\beta^2\right|^{\frac{1}{2}}\sqrt{6}\cdot\cos\frac{\pi}{4}=\sqrt{3}\left|1+\alpha^2+\beta^2\right|^{\frac{1}{2}}=3$$

$$1+\alpha^2+\beta^2=3$$

$$\Rightarrow\alpha^2+\beta^2=2$$

$$|\vec{a}\times\vec{b}|^2=|\vec{a}|^2|\vec{b}|^2\sin^2\frac{\pi}{4}$$

$$=3\times6\times\frac{1}{2}=9$$

$$=2\times9=18$$

Question: If $x\left(x^2+3|x|+5|x-1|+6|x-2|\right)=0$, then the number of solutions of the given equation is ___.

Answer: 1.00

Solution:

$$x\left[x^2+3|x|+5|x-1|+6|x-21|\right]=0$$

$x=0$ or other expression $=0$

But $x^2,|x|,|x-1|,|x-2|$

So only (1) solution

Question: $3\sin(A+B)=4\sin(A-B)$. If $\tan A=k\tan B$, then the value of k is ____.

Answer: 7.00

Solution:

$$\frac{\sin(A+B)}{\sin(A-B)}=\frac{4}{3}\to\frac{2\sin A\cos B}{2\cos A\sin B}$$

$$\tan A=7\tan B$$

$$k=7$$

Question: If $(y-2)^2=(x-1)$ and $x-2y+4=0$, then find the area bounded by the curves between the coordinate axis in first quadrant (in sq. units).

Answer: 5.00

Solution:

$$z=zy-4$$

$$\Rightarrow(y-2)^2=2y-5$$

$$y^2-6y+9=0$$

$$y=3$$

$$\text{Area }=\int_0^2(y-2)^2+1\,dy+\int_2^3(y-2)^2+1-(2y-4)\,dy$$

$$=\frac{(y-2)^3}{3}+y\Big|_0^2+\frac{(y-2)^3}{3}\Big|_2^3+5y-y^2\Big|_2^3$$

$$=2+\frac{8}{3}+\frac{1}{3}+5-5$$

$$=5$$

Question: Find the number of common roots of the equation $z^{1901}+z^{100}+1=0$ and $z^3+2z^2+2z+1=0$

Answer: 2.00

Solution:

$$z^3+2z^2+2z+1=0$$

$$(z+1)\left[\left(z^2-z+1\right)+2z\right]=0$$

$$(z+1)\left(z^2+z+1\right)=0$$

$$\Rightarrow z=-1,\ w+w^2$$

$z^{1901}+z^{100}+1=0$ has w and w^2 as roots

So 2 common roots

Question: $A=\{1,2,3,4\}$. Find numbers of relations which are symmetric but not reflexive.

Answer: 960.00

Solution:

$(1,1)\quad(1,2)\quad(1,3)\quad(1,4)$

$(2,1)\quad(2,2)\quad(2,3)\quad(2,4)$

$(3,1)\quad(3,2)\quad(3,3)\quad(3,4)$

$(4,1)\quad(4,2)\quad(4,3)\quad(4,4)$

Symmetric $= 2^{10}$

Reflexive + Symmetric $= 2^{6}$

Required total $= 2^{10}-2^{6}=1024-64=960$